ADDvocacy: Math Speaks

Across the Curriculum

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Standards Addressed through Project

MA.912.A.10.1: Use a variety of problem-solving strategies, such as drawing a diagram, making a chart, guessing- and-checking, solving a simpler problem, writing an equation, working backwards, and creating a table.

MA.912.A.10.2: Decide whether a solution is reasonable in the context of the original situation.

LA.910.1.6.2: The student will listen to, read, and discuss familiar and conceptually challenging text;.

LA.910.3.1.3: The student will prewrite by using organizational strategies and tools (e.g., technology, spreadsheet, outline, chart, table, graph, Venn Diagram, web, story map, plot pyramid) to develop a personal organizational style.

SS.912.C.2.5: Conduct a service project to further the public good.

SS.912.C.2.10: Monitor current public issues in Florida.

Overview of Project

The source of my idea was the lack of word problem comprehension and fluency among my math students. I was constantly encountering students who were afraid of word problems and/or had no idea how to begin solving one.

Student Impact

Students have scored better on classroom and state assessment. Also, their attitude toward math, in general, is more positive. They have less anxiety when they encounter a word problem.

Project Description

This project improves students' reading comprehension and word problem fluency by having them create word problems that raise awareness about community and global issues. Students are assigned an issue, such as environmental conservation or child safety. Also, students may choose their own cause. Students will research information about the issue to collect numerical facts and statistics related to the issue. Students are taught the 4-step problem solving method using a problem-solving organizer with word problems from traditional curriculum resources like student textbooks. Students are then asked to create their own word problems by filling in blank problem-solving organizers with their own info created using their research and imagination. Alternatively, students may be given other word problems as a template to follow, depending on the math concept that should be included in the student-created word problems. After some practice with problem-solving organizers or templates, students should be able to write word problems from scratch on their own. From this process, students learn how to solve word problems from the inside out. They, in essence, become "word problem psychologists" and are able to identify with what the writers of other word problems are asking them to do. They are also able to advocate for solutions to community and global issues by highlighting them in a math problem that demonstrates accurate statistical data. After students, have created their word problems, they can be showcased to other students and teachers. While there are a variety of ways for students to demonstrate and share their skills and knowledge gained, the following activities have been used in the past: bell ringers for other classes, posters, mini- workbooks, websites and portfolios. The innovative aspect of this project is that it has the ability to peak student interest across the curriculum, not just in math. In addition, students become engaged in helping to solve community issues instead of only being an observer in their community. One of the students who completed this project became an advocate for environmental awareness and confident in his ability to attack word problems because of this project. Prior to the project, he left almost all the word problems on his assignments untouched. When he was asked to make a poster about environmental conservation, he offered to make more and ended up completing three different posters. I have performed several adaptions of this project. It works best when students have to write their own word problems on a regular or ongoing basis.

Student Population

Traditionally, the entire math class has participated in this project, regardless of past achievement levels. Previously the project has been done with students in grades 9-12, but it can be adapted to be used in grades 1-8. For example, younger students can verbally record or illustrate their word problems. This project can be used in both small and large groups.

Materials

The project can be implemented with a low or high level of technology. The project will work with virtually any classroom setup provided there are desktops & seats for students and a whiteboard, overhead or LCD projector for the teacher. High tech adaptions using the Internet, computers and/or a LCD projector are optional. Required materials include problem-solving organizer worksheets, pencils, word problems from book(s) or other sources.

Additional Lesson Resources

Possible lesson resources can include field trips at school and away from school, the media center, Internet, teacher resource books and/or guest speakers. I have been able to implement this project using only a book from a library and the other required materials.

Basic Lesson Plan for ADDvocacy: Math Speaks

Class Objectives:	Students will be able to outline the 4 steps for solving word problems. Students will increase their ability to solve word problems .
Standards:	MA.912.A.10.1: Use a variety of problem-solving strategies, such as drawing a diagram, making a chart, guessing- and-checking, solving a simpler problem, writing an equation, working backwards, and creating a table. MA.912.A.10.2: Decide whether a solution is reasonable in the context of the original situation.
Anticipatory Set:	Show Adopt-A-Tree video Explain that all the people they saw standing in line to get trees and volunteers were interested in planting trees to do their part in “saving the Earth”. Ask students “What’s your cause?”
Introduction:	Explain to students that they will be advocating for their favorite cause using math. Ask the students how many of them have trouble with word problems and let them know that the project also help them understand word problems better.
Procedures:	1. Have students research their cause to gather statistics and other numerical facts 2. Review 4 step problem-solving method with students using word problems from textbook or samples provided (may need a previous lesson depending on grade-level and/or ability) 3. Students will use a modified 4 step problem solving organizer to write their word problems
Conclusion	Students’ word problems should be showcased via products such as bell ringers for other classes, posters, mini- workbooks, websites, animations and portfolios.

Extensions for Service-Learning

Bell ringers for other classes –

Have students create 3-5 word problems to use as a bell ringer to highlight various aspects of a community or global issue. Some issues that could address are energy conservation, breast cancer awareness, animal welfare, teen pregnancy, drug abuse, and teen driver safety. This could be to emphasize observances such as Earth Day or Breast Cancer Awareness Month. Students may also see problems in the school or local community that they would like to make the student aware of.

Posters-

Have students design a poster posing a scenario or question using the statistical data or numerical facts. Posters can be distributed to other schools, community centers, churches, and online.

Mini- workbooks-

Students can combine their efforts to create a mini-workbook to teach younger students about a community or global issue. The mini-workbooks can be used for peer-tutoring or teacher resources.

Websites-

Students can create a website that showcases a collection of word problems about a variety of community and global issues. It can be marketed on teacher resource websites.

Animations-

Students can use Xtranormal, GoAnimate, Toondoo or a similar to create a scenario that uses the statistics and numerical data in a solution to a community or global issue.

The Four-Step Problem Solving Plan Explained

Overview of "Four-Step Problem Solving"

The "Four-Step Problem Solving" plan helps elementary math students to employ sound reasoning and to develop mathematical language while they complete a four-step problem-solving process. This problem-solving plan consists of four steps: details, main idea, strategy, and how. As students work through each step, they may use "graphic representations" to organize their ideas, to provide evidence of their mathematical thinking, and to show their strategy for arriving at a solution.

Main Idea

In this step, the student is a reader, a thinker, and an analyzer. First, the student reads over the problem and finds any proper nouns (capitalized words). If unusual names of people or places cause confusion, the student may substitute a familiar name and see if the question now makes sense. It may help the student to re-read the problem, summarize the problem, or visualize what is happening. When the student identifies the main idea, he or she should write it down, using words or phrases; that is, complete sentences are unnecessary. Students need to ask themselves questions such as the ones shown below.

? "What is the main idea in the question of this problem?"

? "What are we looking for?"

? "What do we want to find out?"

Details

The student reads the problem again, sentence by sentence, slowly and carefully. The student identifies and records any details, using numbers, words, and phrases. The student looks for extra information—that is, facts in the reading that do not figure into the answer. In this step, the student should also look for hidden numbers, which may be indicated but not clearly expressed. (Example: The problem may refer to "Frank and his three friends." In solving the problem, the student needs to understand that there are actually four people, even though "four" or "4" is not mentioned in the reading.) Students ask themselves the following kinds of questions.

? "What are the details needed to answer the question?"

? "What are the important details?"

? "What is going on that can help me answer the question?"

? "What details do I need?"

Strategy

The student chooses a math strategy (or strategies) to find a solution to the problem and uses that strategy to find the answer/solve the problem. Possible strategies include the following.

- use or draw a picture
- look for a pattern
- write a number sentence
- use actions (operations) such as add, subtract, multiply, divide
- make or use a table
- make or use a list
- work a simpler problem
- work backwards to solve a problem
- act out the situation

The preceding list is just a sampling of the strategies used in elementary mathematics. There are many strategies that students can employ related to questions such as the following.

? "What am I going to do to solve this problem?"

? "What is my strategy?"

? "What can I do with the details to get the answer?"

How

To make sure that their answer is reasonable and that they understand the process clearly, students use words or phrases to describe how they solved the problem. Students may ask themselves questions such as the following.

? "How did I solve the problem?"

? "What strategy did I use?"

? "What were my steps?"

In this step, students must explain the solution strategy they have selected. They must provide reasons for and offer proof of the soundness of their strategy. This step gives students the opportunity to communicate their understanding of math concepts and math vocabulary represented in the problem they solved and to justify their thinking.

Responses on these four parts need not be lengthy—a list of words and numbers might be used for the details, and phrases might be used for the "Main Idea" and "How."

Benefits of Using "Four-Step Problem Solving Plan"

One of the method's major benefits to students is that it forces them to operate at high levels of thinking. Teachers, using the tried-and-true Bloom's Taxonomy to describe levels of thinking, want to take students beyond the lower levels and help them reach the upper levels of thinking. Doing the multiple step method requires students to record their thinking about three steps in the process, in addition to actually "working the problem."

A second benefit of extending the process from three steps to four is that having students think at these levels will deepen their understanding of mathematics and improve their fluency in using math language. In the short term, students' performance on assessments will improve, and confidence in their mathematical ability will grow. In the long term, this rigor in elementary school mathematics will prepare students for increased rigor in secondary mathematics, beginning particularly in grade 7.

Another benefit of using "Four-Step Problem Solving" is that it will increase teachers' ability to identify specific problems students are having and provide them with information to give specific corrective feedback to students.

Extracting and writing the main idea and details and then showing the strategies to solve problems should also help students establish good test-taking habits for online testing.

Educational Research Supporting "Four-Step Problem Solving"

Although scholarly articles do not mention "Four-Step Problem Solving" by name, most educational experts do advocate the use of multi-step problem-solving methods that foster students' performing at complex levels of thinking. The number of steps often ranges from four to eight.

Conclusions drawn from studying the work of meta-researcher Dr. Robert Marzano published in the book Classroom Instruction That Works (Marzano, Pickering, Pollock) as well as numerous other research studies, indicate that significant improvement in student achievement occurs when teachers use these strategies.

Instructional Strategy	Average **Percentile Gain**	Relationship to **"Four-Step Problem Solving"**
Summarizing and Note-taking	34 points	Main Idea, Details, How
Arguing (in the sense of defending or justifying one's thinking)	29 points	How
Articulating generalizations and principles	29 points	How
Providing feedback to students ("corrective," timely, specific)	29 points	Grading
Using nonlinguistic representations	27 points	Strategy
Using advance (graphic) organizers	22 points	All Steps

The National Council of Teachers of Mathematics endorses the use of such strategies as those appearing in “Four-Step Problem Solving”—particularly the step requiring students to explain their answers—as effective for producing students’ math competency, as described in NCTM publications such as Principles and Standards for School Mathematics. Excerpts from NCTM documents validate the district's problem-solving strategy. Some of the key ideas and teaching standards identified include the following.

- Teachers need to investigate how their students arrive at answers. Correct answers don't necessarily equate to correct thinking.
- Students need to explore various ways to think about math problems and their solutions.
- Students need to learn to analyze and solve problems on their own.
- Students' discourse in a mathematics classroom should focus on their thinking process as they solved a problem.

Service-Learning Definition, Elements, and Examples

Definition

Service-learning is a method by which students learn and develop through active participation in thoughtfully organized service that

- Is conducted in and meets real needs of a community (schools may be defined as community),
- Is integrated into and enhances the academic curricula of students,
- Provides structured time for students to reflect on their service experiences and demonstrate knowledge or skills they have gained, and
- Helps foster civic responsibility.
-

In service-learning projects, students practice skills and behaviors that teachers need them to learn through the service they do. Service is a means and application of learning.

Student Components in Service-Learning Projects

What distinguishes service-learning from other service and volunteering? In a school context, the service is directly related to curricula and components of the project are course assignments and part of students' grades. Activities are designed to address and apply specific learning objectives, standards, and curriculum frameworks. Effective service-learning projects have the following student roles:

<u>Needs Investigation and Project Design</u>

- Needs identification/assessment—those being served help define their needs and how to meet them.
- Learning the context for the need(s) to be addressed.
- Issue discussion and selection.
- Examination of stakeholders, policies, and systems impacting the need(s) to be addressed.
- Designing activities to meet identified needs.

Teachers assign students work/projects/tasks to learn about the context for the service the students will subsequently provide. The service that follows is derived from this new knowledge, involves student voice and design, but remains within the curricular framework the teacher has established.

<u>Action</u>

- Research- and knowledge-based service activities.
- Student leadership in conducting and leading project activities.
- Activities are often fluid and evolve as the project progresses, original needs are addressed, and others are identified.
- Students work collaboratively with service recipients and partners.
- Application of multiple learning styles, including individual work, teamwork, using technology, tactile/manual work, oral presentations, data collection, writing, construction, etc.
- Activities flow from preparation and are not wholly pre-planned by teachers.

Reflection

Reflection is integrated into successful projects from beginning to end as students form and test opinions, project outcomes, measure results and impacts, discuss actions and reactions, and make improvements and future plans. Reflection allows students to process and absorb what they have experienced and is critical to meaningful learning.

Reflective activities include the following:

- Journaling,
- Projecting project impacts,
- Discussion,
- Conducting formative and summative evaluation,
- Making project refinements, and
- Conducting future planning.

Demonstration

Demonstration is another application of service-learning that involves students in educating others about the issues they are addressing.

Demonstration takes various forms—many of which are actual service-learning projects in themselves—including the following:

- Advocacy campaigns,
- Putting on public forums/presentations,
- Performance on the service issues,
- Teaching others about the project and the issues behind it (lessons, presentations), and
- Creating films, portfolios, books, Web sites, publications, works of art, etc.

Recognition/Celebration

Throughout the project, but especially at the end, students should be recognized for their efforts. In successful projects, all participants join together to reflect and to plan future efforts.

Youth Empowerment

An implicit or explicit component of the above elements, youth empowerment and leadership enrich every aspect of service-learning. The greater the voice students have in identifying needs and designing activities, the more motivated they will be. Having to demonstrate to (i.e., teach) others about the needs and issues being addressed requires a higher assimilation of learning. Effective projects have students evaluate impacts of activities, saving teachers labor and making students responsible for determining whether their efforts were successful. Strong student roles are hallmarks of effective projects, in which students . . .

- Are involved in project design and planning, with meaningful leadership roles (including needs identification and helping to decide what service activities will be conducted). Teachers in effective projects assign students organizing and logistical duties involved in arranging, providing, measuring, evaluating, reporting, and celebrating service activities.

- Conduct a lot of service over time. It is better to have fewer students doing a lot of service-learning than to have a lot of students conduct only surface-level efforts.

Reciprocity

Reciprocity ensures activities provide service that is actually needed, exposes students to people different from themselves, provides different perspectives on needs, and brings outside assistance, expertise, match, and publicity, to service-learning projects. In effective projects, teachers and students design activities based on what they hear and learn from those in need and not on preconceived notions.

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When these elements are in place, the impacts of curriculum-based service-learning go far beyond those of traditional community service and volunteering. Service-learning combines academic and affective learning to engage students hands-on in the real world. This combination—not to mention the incentive to learn outside the classroom—is what makes service-learning such a powerful tool, pedagogy, and strategy.

Examples of Service-learning in Various Need Areas

Reading—e.g., students serving as reading tutors for other students or for adults; creating books or other written materials for other students, the public, or Web sites; reading and writing for seniors or the infirm; editing documents; teaching reading to young children; translating documents for non-English speakers; promoting reading through advocacy campaigns, public service announcements (PSAs), book drives, or public readings; designing and constructing reading areas; and dramatic, artistic, or musical performances of texts and literature.

Civics/history—e.g., conducting, compiling, recording, publishing, filming, or depicting histories of a local community, individuals in a community, or historic locations (cemeteries, buildings, natural features/sites, forts, Native American sites); advocacy campaigns on topics in the public interest; gathering and disseminating information about services available to residents and visitors; creating murals depicting local history; teaching peers about democratic processes through events, student-made videos, performances (including puppet shows), lessons, and hands-on activities; creating children's history books; serving as museum docents; reenacting historic events; restoring or recreating historic structures; forums on topics of public interest; oral histories focusing on different eras; teaching about voting; producing tip sheets or guidebooks on how to effect positive community change.

Drug/violence prevention—e.g., teaching other students or the community how to avoid/respond to conflict, drugs, STDs, teen pregnancy, alcohol, and other self-destructive choices. Strategies could include lessons, presentations, dramatic performances, videos, artistic displays, music, advocacy campaigns, PSAs, forums, coloring books, conflict mediation, serving on Teen Courts, and safety presentations for the home, car, or neighborhood.

Intergenerational interaction—e.g., service projects for and with seniors to include health screenings, exercise programs, teaching the use of computers, oral histories, pen pal programs,

concerts and dances with (not just for) seniors, creating art or gardens at senior centers, working with seniors to put on public forums on important issues, and providing patients with physical and mental stimulation (working on arts and crafts together, exercise, games, etc.). Students can also teach others about seniors through lessons, publications, presentations, performances, brochures, Web sites, and advocacy campaigns.

Environment—e.g., restoration of degraded areas; exotic plant removal; propagation and planting of native plants; water, flora, and fauna testing/monitoring; research on endangered species; erosion abatement efforts; management of public lands to include trail and outdoor classroom design and maintenance; raise-and-release efforts; energy audits for homes, schools, and communities; and mapping. Demonstration elements include teaching, presenting, creating brochures and Web sites, art representing the flora and fauna being studied, giving tours and field days, making videos, composing information to place in kiosks and translating it into foreign languages, performances, advocacy campaigns, public service announcements, Web sites, and fundraising to preserve natural areas.

RESOURCES

Resources can be found on pages 16-43

Name: _____ Date: _____ Class #:_____

4 Steps to Problem Solving Organizer

Understand What do you know?	What do you need to find out?

Problem:_____

_____

_____

Plan

*Set up an equation or ratio *Make an organized list * Make a chart or table
*Work backward *Draw a picture or diagram *Use a model or objects
*Guess and check *Look for a pattern *Solve a simpler problem *Act it out

First I will…

Next I will…

Then I will…

Solve

Show ALL work.
Number each step.
Label all numbers with units.

Look Back

Check your work.
Does it make sense?
Does it answer the question?

***Challenge!** Write your own problem similar to the one you just solved and have a partner try to solve your problem.

Name: _____ Date: _____ Class #:_____

4 Steps to Problem Writing Organizer

Understand What will the student need to know to solve your problem?	What will the student need to find out? What info will you need to give the student for them to find out this info?

Write a Problem (using statistics & numerical values from your research and responses above):

_____

_____

_____

_____

Plan

*Set up an equation or ratio *Make an organized list * Make a chart or table
*Work backward *Draw a picture or diagram *Use a model or objects
*Guess and check *Look for a pattern *Solve a simpler problem *Act it out

First student will…

Next student will…

Then student will…

Solve

Show ALL work.
Number each step.
Label all numbers with units.

Look Back

Check your work.
Does it make sense?
Does it answer the question you posed?

WOW! Writing In Math and Problem Solving

Words

CIRCLE important words in the question

IDENTIFY math vocabulary to use

Organize ideas into a...

Table **Chart** **Picture with labels** **List**

Write sentences

- ✓ **Use** important words **from question to state the main idea**
- ✓ **Use** identified **math vocabulary to add supporting details**
- ✓ **Be sure to include equations (number sentences) in your explanation**
- ✓ **Is what you wrote clear to the reader?**

Name: _____ **Date:** _____ **Class:** _____

Words

- ☐ **Circle important words**
- ☐ **Identify math vocabulary to use**
- ☐ **What do I need to find out? (Underline the question):** _____

_____

_____

Organize ideas

- ☐ **Show your work in a table, chart, picture with labels, list, or diagram to prove your solution. Circle your answer.** *(Don't forget titles, labels, keys, etc.)*

Write sentences

- ☐ Use *important words* from the question to state the main idea
- ☐ Use *identified* math vocabulary to add supporting details
- ☐ Be sure to include equations (number sentences) in your explanation
- ☐ Is what you wrote clear to the reader?

_____

_____

_____

_____

_____

_____

_____

_____

** How else could I have solved this problem? _____

_____

_____

_____

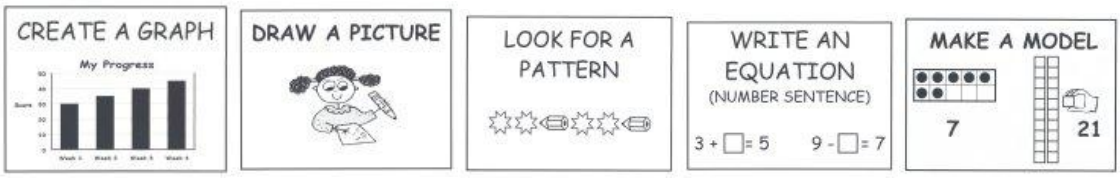
Problem Solving Practice

Read and Understand

Look at the problem above. Underline the question that you will answer.

Plan

Circle the strategy or strategies that will help you solve the problem.



Solve

Solve the problem. Use the strategy or strategies that you circled to help you.

Use what you know about _____ to explain how you solved the problem. Use words, pictures and/or numbers in your explanation.

_____

_____

_____

_____

Look Back

- **Did I answer the question?**
- **Does my answer make sense?**
- **Did I check for accuracy?**

4 Steps to Problem Solving Checklist

Name: _____ Date: _____ Class: _____

4 Steps to Problem Solving!

1.) UNDERSTAND THE PROBLEM

____ Read the problem carefully (1st time) with NOTHING in your hands

____ Read the problem again and...

- ____ Underline important information
- ____ Circle important words and math vocabulary
- ____ Highlight the question

____ *Write a sentence clearly stating the problem*

____ *List all important information needed*

____ *Write the units you will use to LABEL YOUR ANSWER*

2.) PLAN (*I will...*)

____Set up an equation or ratio

____Make a chart or table

____Draw a picture or diagram

____Guess and check

____Solve a simpler problem

____Make an organized list

____Work backward

____Use a model or objects

____Look for a pattern

____Act it out

3.) SOLVE THE PROBLEM

____ *Carry out all steps and show all work*

- Number each step
- Label the numbers with units or tell what they represent
- Show your work neatly within each step
- If using a calculator, write the equation and answer
- Label diagrams
- Add any information needed to inform the reader

____ *Write a solution sentence and correctly label the solution*

4.) LOOK BACK & REFLECT ON WORK

____ Check all math work

____ Is each step clearly shown & labeled?

____ Is your answer reasonable and has the correct units?

____ Did you answer the question?

____ Revise strategies and work as needed

____ Check revised work

Math connection...

____ Solve the problem another way to check the solution

____ Describe how you might solve this type of problem differently next time & why?

____ Have you solved a similar problem before? How did that influence your work in this problem?

____ Have you had an experience in real life like this problem? Explain.

PROVE IT! JUSTIFY IT! PROVE IT! JUSTIFY IT!

Here is my answer. _____

I feel confident that it is correct because _____

_____

_____

_____

Here are the equations that support my answer. This is my mathematical proof!

+-x÷+-x÷+-x÷+-x÷+-x÷+-x÷+-x÷+-x÷+-x÷+-x÷+-x÷+-x÷+-x÷

I am ready to defend the accuracy of my solution!

Signed-_____

What Good Problem-Solvers Do ...

In the ***Read and Understand*** section:

- ✓ Highlight or circle key words and numbers in the story problem.
- ✓ Re-read the story problem as many times as necessary to understand.
- ✓ Make a reasonable estimate of the answer BEFORE actually solving the problem.

  Think … what is the lowest estimate? ("Floor")

  What is the highest estimate? ("Ceiling")

- ✓ Connect words in the problem to mathematical operations.

  Some examples:

  Subtract ~ remove, spend, buy, give away, how many more

  Add ~ combined, altogether, increase, total, how many in all

In the ***Plan*** section:

- ✓ Choose the strategy or strategies that will help solve the problem quickly and easily.

In the ***Solve*** section:

- ✓ Label the numbers and math work in the solution area.
- ✓ Use an inverse operation to check the computation.
- ✓ Compare the actual answer to the estimate made before --- Is the answer close to the estimate? Is the answer reasonable?

In the ***Look Back*** section:

- ✓ Before writing, make a word "splash" or word bank of math vocabulary related to the problem, in the margin beside the written response area.
- ✓ Use mathematical words, not equations, to share how a problem was solved or why the answer is correct.

  Example: Instead of – *I did 328 + 403 and got 731,* write – *I added 328 books and 403 books and got a sum of 731 books.*

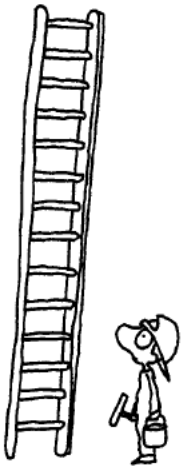
- ✓ Use numbers from the story problem in the written response.

UNIT P3 ■
Problem Solving
Middle Primary

Blake's Topic Bank

Problem Solving
Acting it Out or Using Concrete Material

by Sharon Shapiro



This unit contains:

- Teaching notes
- 3 teaching examples
- 1 BLM
- 15 task cards
- Answers

Problem Solving

Acting it Out or Using Concrete Materials

Sharon Shapiro

Middle Primary

The Problem Solving Process

It is important that students follow a logical and systematic approach to their problem solving. Following these four steps will enable students to tackle problems in a structured and meaningful way.

Step 1: Understanding the problem

- Encourage students to read the problem carefully a number of times until they fully understand what is wanted. They may need to discuss the problem with someone else or rewrite it in their own words.
- Students should ask internal questions such as, what is the problem asking me to do, what information is relevant and necessary for solving the problem.
- They should underline any unfamiliar words and find out their meanings.
- They should select the information they know and decide what is unknown or needs to be discovered. They should see if there is any unnecessary information.
- A sketch of the problem often helps their understanding.

Step 2: Students should decide on a strategy or plan

Students should decide how they will solve the problem by thinking about the different strategies that could be used. They could try to make predictions, or guesses, about the problem. Often these guesses result in generalisations which help to solve problems. Students should be discouraged from making wild guesses but they should be encouraged to take risks. They should always think in terms of how this problem relates to other problems that they have solved. They should keep a record of the strategies they have tried so that they don't repeat them.

Some possible strategies include:

- Drawing a sketch, graph or table.
- Acting out situations, or using concrete materials.
- Organising a list.
- Identifying a pattern and extending it.
- Guessing and checking.
- Working backwards.
- Using simpler numbers to solve the problem, then applying the same methodology to the real problem.
- Writing a number sentence.
- Using logic and clues.
- Breaking the problem into smaller parts.

Step 3: Solving the problem

- Students should write down their ideas as they work so they don't forget how they approached the problem.
- Their approach should be systematic.
- If stuck, students should reread the problem and rethink their strategies.
- Students should be given the opportunity to orally demonstrate or explain how they reached an answer.

Step 4: Reflect

- Students should consider if their answer makes sense and if it has answered what was asked.
- Students should draw and write down their thinking processes, estimations and approach, as this gives them time to reflect on their practices. When they have an answer they should explain the process to someone else.
- Students should ask themselves 'what if' to link this problem to another. This will take their exploration to a deeper level and encourage their use of logical thought processes.
- Students should consider if it is possible to do the problem in a simpler way.

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Teaching Notes Acting it Out or Using Concrete Materials

Sometimes it can be hard for students to gain a concrete understanding of an abstract problem. To assist students who are finding it difficult to visualise a problem, or the procedure necessary for its solution, it is often helpful to use objects (concrete materials) to represent the people or things in the problem. A variety of objects such as counters, blocks, pencils or rubbers can be used to symbolise people or places. These objects can be moved through the steps of the problem. It is important to chart this movement to keep track of the process.

It can also be very helpful for students to act out the roles of the different participants in the problem.

Certain skills and understandings should be reinforced before students begin to work with this strategy.

Moving from one position to another

If the characters, or objects, featured in a problem move around a lot, it can be confusing and difficult to solve. By getting students to act out the problem, or using objects to act it out, the movements in the problem can be plotted.

For example: The window cleaner was standing on the middle rung of the ladder cleaning the outside windows of the office block. He climbed up three rungs to clean some windows then saw a spot he had missed below him. He climbed down seven rungs to clean it and then climbed up the remaining ten rungs. He was now at the top of the ladder. How many rungs were there altogether on the ladder?

Now ask the students to act out the problem. Draw a chalk ladder on the ground making each rung an equal distance apart. Indicate which direction is up the ladder with a chalk arrow, and mark the middle rung of the ladder. Choose a student to be the window cleaner. He or she should start by standing in the middle of the ladder. The student should walk for three rungs towards the top of the ladder. Then ask the student to turn and walk 'down' for seven rungs. Then turn and walk 'up' for ten rungs and mark that spot as the top of the ladder. Ask the students to count how many rungs there are back to the middle point. By counting the same number of rungs below the middle point, the students can calculate that the ladder has 13 rungs.

A similar approach can be taken with the following problem. A spider is climbing up the side of a ten metre wall. Each hour it climbs three metres and then rests for one hour. During its rest the spider slips back one metre. How long till it reaches the top of the wall?

Draw a ten metre line on the ground and use students, or an object, to show the movements of the spider. Ask other students to count one hour for every three metres the spider moves forward and one hour for each metre it slips back while resting.

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Teaching Notes Acting it Out or Using Concrete Materials

Amounts of money

Often amounts of money, or quantities of substances, are exchanged between the characters in a problem. Unless a means of visualising or acting out the problem is used, the result can be very confusing. This method is particularly helpful with more complex exchanges where students may not be able to write or explain the operations accurately.

For example: Their grandparents sent Nick, Sally and Shirley $160, in total, for their birthdays. Their parents had to divide the money up so that Nick was given $20 more than Sally and $30 more than Shirley. How much were they each given?

Ask three students to act out the parts of the children and use $160 play money for the exercise.

Start by giving Sally an estimated amount.

Give Sally $40.

Nick should be given $20 more, $40 + $20 = $60

Shirley should be given $30 less than Nick, $60 – $30 = $30

Total $40 + $60 + $30 = $130. This total is too low.

Start with a higher amount.

Give Sally $50

Nick should be given $50 + $20 = $70

Shirley should be given $70 – $30 = $40

Total $50 + $70 + $40 = $160. This is correct.

Specific quantities

Sometimes a problem requires students to measure out an exact quantity, but they do not have access to containers which can hold the correct amount. The students must come up with a way to accurately measure the required quantity using the containers they have. Acting out the process makes it easier to work out a solution.

For example: If exactly three litres have to be poured into a container, and I only have a two litre and a five litre container, what can I do?

I can fill a five litre container with water and pour off exactly two litres into the two litre container. Then I am left with three litres in a container. The correct amount remains.

Using concrete materials

When a problem contains large numbers (of objects or people) it may not be practical to use students to act it out. Using concrete materials, such as counters or blocks, will assist students to work through the process.

For example: John is queuing at the canteen. There are 50 people in front of him, but he is very impatient. Each time a student in the front is served, John slips past two students. How many people will be served before John reaches the front of the line?

It will help students to visualise the movement in the problem by actually moving the counters or blocks to each new position.

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Teaching Examples

Acting it Out or Using Concrete Materials

Example 1

Two adults and two children have been stranded on an island in a river. They must cross the wide river to safety, but they only have one canoe. The canoe can either take one adult or two children at a time. How can they safely reach the other side?

Understanding the problem

What do we know?

There is one canoe.
It can hold one adult or two children at a time.
There are two adults and two children.

What do we need to find out?

Questioning: How can they safely leave the island? How many trips will it take?

Planning and communicating a solution

What we did

Students should label a block for each member of the family – A for each adult and C for each child. Demonstrate for the students how to write down each step, as below, so that steps are not duplicated.

Students should create three areas. One for the island, the second for crossing the river and the third for the safe river bank.

Remaining on the island	Crossing the water	Safe river bank
AACC on the island		
AA remain	→ CC →	CC
AAC	← C	← C
AC	→ A →	AC
ACC	← C	← A
A	→ CC →	ACC
AC	← C	← AC
C	→ A →	AAC
CC	← C	← AA
	→ CC →	AACC

Reflecting and generalising

Using blocks to represent the movements of the family members made it easier to work through the steps of the problem and arrive at an accurate answer.

Extension

What if there were five people on the island instead of four (three children and two adults)? What if an adult and a child could fit in the canoe at the same time? How would this affect the speed of moving people off the island?

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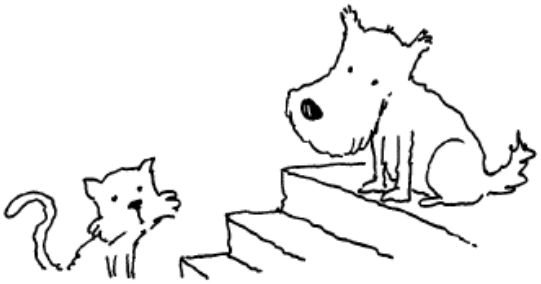
Example 2

In this example counters, blocks or students could be used to symbolise the characters in the problem.

Blackie the cat was lying asleep on the middle step and the dog Kuja arrived and sat three steps above him. There were two steps between the dog and the top step. How many steps were there altogether?



Understanding the problem

What do we know?

Blackie was lying on the middle step.
Kuja sat three steps above.
There were two steps between Kuja and the top step.

What we have to find out?

Questioning: How many steps were there altogether?
How many steps were there below Blackie?

Planning and communicating a solution

What we did

Draw a staircase or make one out of blocks. Use a counter to show Blackie's position in the middle of the staircase.

Place a counter to represent Kuja, three steps above the Blackie counter. Leave two steps empty and then the third step should be the top step. Count with students to find out how many steps there are above Blackie and add the same number below Blackie's position. There are six below and six above plus the middle step, so there are 13 steps altogether.

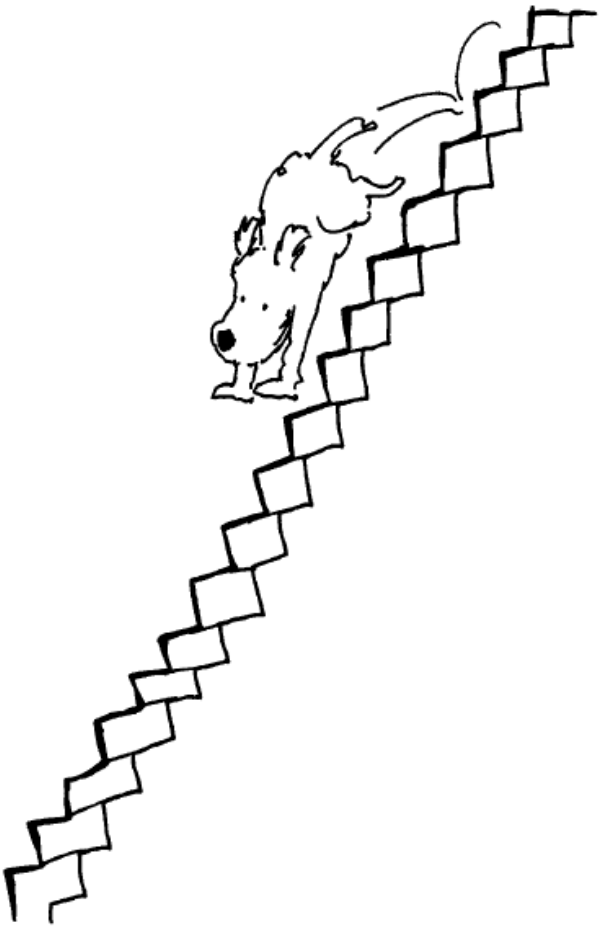
Students can also act out the problem on some real steps or a flight of stairs.

Reflecting and generalising

The problem is much easier to calculate once steps are made or drawn. Then it is simply a matter of counting the steps above Blackie and adding the same number below, plus the step Blackie is sitting on. The answer is concrete and accurate.

Extension

The problem can be extended by including additional animals or by moving animals to different steps. What if Blackie's sister Poppy came and sat three steps above Kuja and then moved to the step second from the bottom? How many steps below Blackie would she be? How many steps did she move when she changed position?



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Example 3

A class contained 32 students. The students were standing in a circle and began to count round the room starting from one, with each student saying one number. As they counted, all those who said even numbers sat down. Once each person had a turn, they continued counting (from 33) using only those students still standing. Again, every student who said an even number sat down. After the second round of counting how many students were still standing? (This problem can be done with different numbers of students, as long as the number is even.)

Understanding the problem

What do we know?

There are 32 students all counting one number.
If a student calls out an even number they sit down.
Two rounds of counting are completed.

What do we need to find out?

Questioning: How many students will still be standing after a second round of counting?

Planning and communicating a solution

Students need to think of a way to visualise the class. They can ask the whole class to stand and act out the problem. If working individually or in pairs, they can use tally marks on paper, or have 32 blocks to represent the class.

As each number is said, the students should sit down if they say an even number, or if they are using paper, cross out the even marks, or remove the even blocks. This should be done for two rounds of counting. They should then count how many students are still standing at the end. There were 8 students still standing at the end. After two rounds of counting we found half of a half, which is a quarter.

Reflecting and generalising

This problem is difficult to visualise because of all the changing factors. If students draw, create or use themselves in the problem the solution is easy to work out.

Extension

This strategy can be used to reinforce all tables, square numbers or powers of numbers.

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BLM Acting it Out or Using Concrete Materials

★ Understanding the problem

List what you know .....

.....

★ What do you need to find out?

Questioning : What questions do you have? What are you uncertain about? Is there any unfamiliar or unclear language? What are you being asked to do?

.....

.....

.....

★ Planning and communicating a solution

Which solution will you try? Are you moving from one position to another? Are you exchanging amounts of money? Are you trying to work out specific quantities without the use of appropriate containers? Will you use tally marks or objects to stand for things, or will people act out the problem? Explain what you did using mathematical language.

★ Reflecting and generalising

How accurate is your answer? How can this strategy be applied to other situations? Could a more effective method have been used? What technology was useful? .....

.....

.....

★ Extension

How can this problem be extended? What factors can be added as part of a 'what if' question?

.....

.....

.....

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PROBLEM SOLVING TASK CARDS

Acting it Out or Using Concrete Materials

Problem 1

Space

Level 1

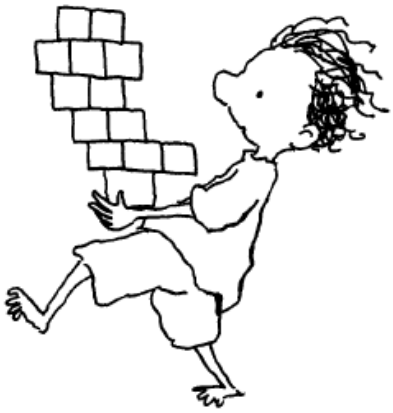
Five blocks have each been labelled with one of these letters – H, I, J, K, L. The H block is immediately to the right of the I block. The J block is to the right of the K block. The I is between the L and K. The L is immediately to the left of the I. Where is the K?

Problem 2

Space

Level 1

Place 14 blocks in three piles. The first pile should have one less than the third. The third pile should have twice as many as the second. How many blocks are there in each pile?



Problem 3

Measurement

Level 1

Four students measured their heights. Nicky was taller than Kelly, but not as tall as Tali. Gene was taller than Tali. Write down their names in order of their heights, from shortest to tallest.

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Problem 4

Measurement

Level 1

George has a one litre bottle of lemonade. He pours half of the lemonade out into a jug, then gives the bottle to Jenny. Jenny is pours half of what's left in the bottle into two large tumblers. Then she hands the lemonade bottle to James. James pours half of what's left in the bottle into a plastic cup. How much lemonade is left in the bottle? What fraction of a litre is this?

Problem 5

Number 1 2 3

Level 1

At a party every guest shakes hands with every one of the other 12 guests. How many handshakes were there?

Problem 6

Number 1 2 3

Level 1

Twenty nine students were waiting in a line to play a game. The teacher chose the first person in the line and then every fourth person in the line after that. How many were chosen?

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Problem 7

Space

Level 2

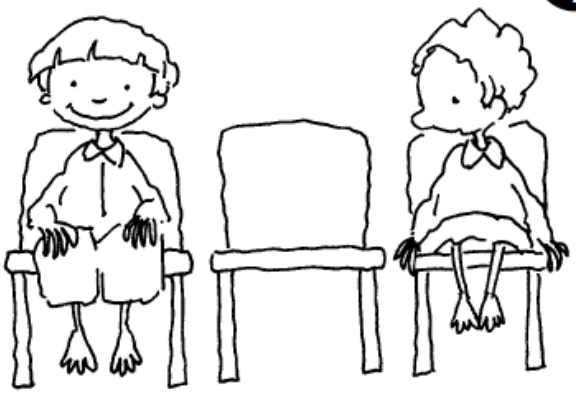
Mrs Peters has four squares of yellow cardboard. She asks her class to join them all so they are connected by at least one edge? How many different ways can you do this?

Problem 8

Space

Level 2

Five students are sitting in a row of chairs along one side of the room. Jennifer sits beside Alison but not beside Peta. Steven sits in the second seat on the left. Alison sits between (but not necessarily next to) David and Peta. Steven sits beside David. Who sits in the middle seat?



Problem 9

Measurement

Level 2

At the water tap there are only three litre and five litre containers. The children must pour exactly seven litres of water into a tub. How can they measure exactly seven litres?

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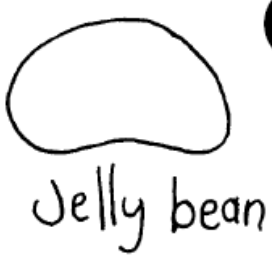
Problem 10

Number 1 2 3

Level 2



Each time a student did something helpful, Mrs Turner removed one jelly bean from container B and placed it in container A. If a student did something unhelpful she did the reverse.

Container A had more jelly beans than container B. Mrs Turner took three jelly beans from container A and put them in container B. Then from container B she took two jelly beans to put in container A. Both containers then had eight jelly beans. How many were in each container in the beginning?

Problem 11

Number 1 2 3

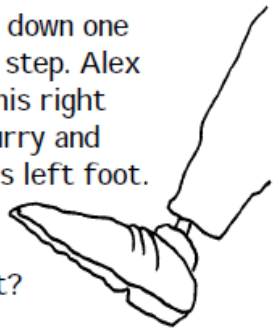
Level 2

Imagine that you bought a game for $15, then sold it for $20 because you no longer played with it. Then you bought it back for $25 because your sister wanted it. Finally, when you sister got bored with the game, you sold it for $30. How much money did you make or lose?

Problem 12

Space

Level 3



Three children walk down a fifteen step fire escape. Lisa walks down one step at a time. She begins by putting her left foot on the first step. Alex is in a hurry and walks down two steps at a time, starting with his right foot. He starts on the second step. Joel is in even more of a hurry and takes three steps at a time, beginning on the third step with his left foot.

Which step will be the first one they all step on?
Will all the children stand on the same step with their left foot?

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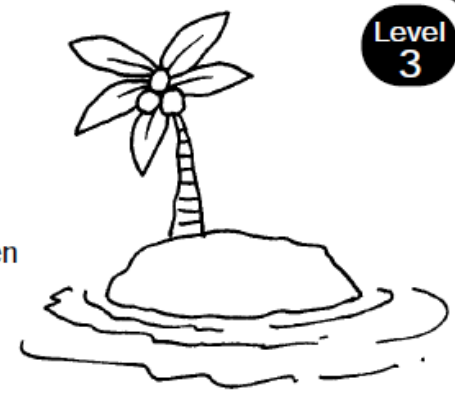
Problem 13

Space

Level 3

An extended family was stranded on an island by freak floods. There were four adults and two children who needed to get off the island in a small rowing boat. The boat can carry two children or one adult. How many trips does the boat have to take in order to carry everyone to safety?



Problem 14

Measurement

Level 3

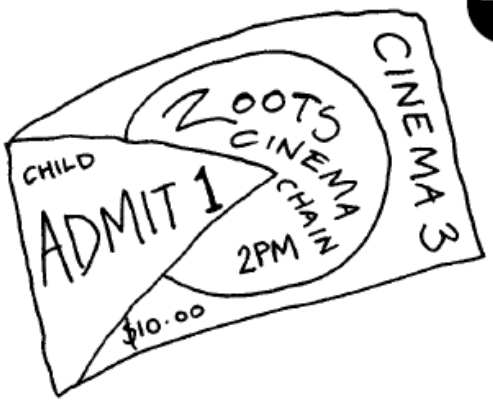
A woman wants to collect exactly four litres of water from the well for her family. She only has two containers, one that can carry five litres and one that can carry seven litres. How can she measure out exactly four litres or water?

Problem 15

Number 1 2 3

Level 3

The same number of seats has been placed in each row at the theatre. Mrs Abrams chair is in the third row from the front, and the eighteenth row from the back. Mrs Dave's seat has eight chairs to the left of it and eleven chairs to the right. How many chairs are there in the theatre?



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Answers to Task Cards

Problem 1

K is the second block from the right.

L	I	H	K	J

Problem 2

The first pile has five blocks, the second pile three blocks and the third pile six.

Problem 3

The order of the students' heights, from shortest to tallest, is Kelly, Nicky, Tali, Gene.

Problem 4

The lemonade bottle has 125 ml left in it.
This is $\frac{1}{8}$ of a litre.

Problem 5

There were 66 handshakes.

The first person shook eleven hands, the second person only needed to shake ten hands, because she had already shaken the hand of the first person.

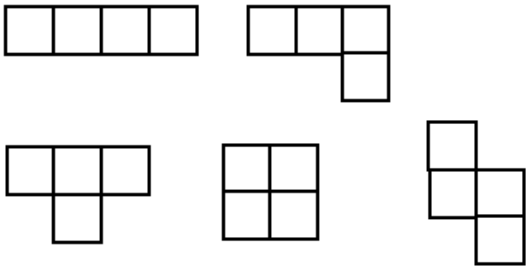
The third person shook nine hands, and so on. So,
11 + 10 + 9 + 8 + 7 + 6 + 5 + 4 + 3 + 2 + 1 = 66

Problem 6

Eight students were chosen.

Problem 7

You can connect the four yellow squares of cardboard five different ways.



Problem 8

Jennifer sits in the middle seat.

David, Steven, Jennifer, Alison, Peta

Problem 9

The children should fill the five litre container with water. Then they should pour three of the five litres of water into the three litre container leaving exactly two litres in the five litre container. They should pour these two litres into the tub and then refill the five litre container and pour that into the tub. There will now be seven litres of water in the tub.

Problem 10

Container A started with nine jelly beans and container B started with seven jelly beans.

Problem 11

You made $10 profit.

Buy	Sell	Profit	Loss
$15			– $15
	$20	+ $5	
$25			– $20
	$30	+ $10	

Problem 12

1. They will all step on the sixth step with their right foot.
2. No, they do not all stand on the same step with their left foot.

	Lisa	Alex	Joel
1	L		
2	R	R	
3	L		L
4	R	L	
5	L		
6	R	R	R
7	L		
8	R	L	
9	L		L

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	Lisa	Alex	Joel
10	R	R	
11	L		
12	R	L	R
13	L		
14	R	R	
15	L		L

Problem 13

It will take 17 trips to rescue the family from the island.

Island		Crossing river		Safety
(A1 A2 A3 A4 CC)				
A1 A2 A3 A4	→	CC	→	CC
A1 A2 A3 A4 C	←	C	←	C
A1 A2 A3 C	→	A4	→	A4 C
A1 A2 A3 CC	←	C	←	A4
A1 A2 A3	→	CC	→	A4 CC
A1 A2 A3 C	←	C	←	A4 C
A1 A2 C	→	A3	→	A4 A3 C
A1 A2 CC	←	C	←	A3 A4
A1 A2	→	CC	→	A3 A4 CC
A1 A2 C	←	C	←	A3 A4 C
A1 C	→	A2	→	A2 A3 A4 C
A1 CC	←	C	←	A2 A3 A4
A1	→	CC	→	A2 A3 A4 CC
A1 C	←	C	←	A2 A3 A4 C
C	→	A1	→	A1 A2 A3 A4 C
CC	←	C	←	A1 A2 A3 A4
	→	CC	→	A1 A2 A3 A4 CC

Problem 14

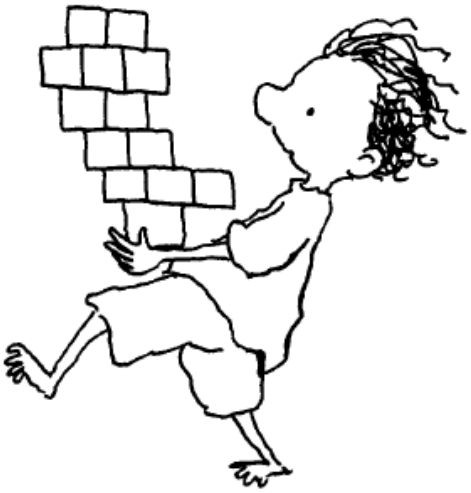
The woman should take the seven litre container and fill it with water. Then she should pour five litres of water from the seven litre container into the five litre container. This will leave two litres remaining in the seven litre container. She should then empty the five litre container and pour into it the two litres from the seven litre container. She should fill the seven litre container again and then pour from the seven litre container into the five litre container (which still has two litres of water in it) until it is full. This will leave exactly four litres in the seven litre container.

Problem 15

The theatre has 400 seats
(20 across and 20 from front to back).

Front 1 2 [3] 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 Back
Mrs Abrams

Left side 1 2 3 4 5 6 7 8 [ ] 1 2 3 4 5 6 7 8 9 10 11 Right side
Mrs Dave



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Videos

Problem Solving song (cartoon)

http://youtu.be/EW2083hpq4Y

Problem solving-students sing steps

http://youtu.be/0RAKtY43gEg

Sources

4-step Problem Solving Explained

http://www.cfisd.net/dept2/curricu/elmath/4step.htm

4 step problem solving organizers

http://br.skschools.net/teachers/dcouture/00BE11EF-011EDE84.6/4%20steps%20to%20problem%20solving%20organizer.pdf

http://br.skschools.net/teachers/dcouture/Graphic%20Organizers

Problem-Solving: Acting It Out or Using Concrete Materials

https://www.blake.com.au/v/vspfiles/.../PT3_ProblemSolving.pdf

Definition of Service Learning

http://education.ufl.edu/learnandserve/sl/sl.html